MATH 010–Linear Algebra Spring 2026
31595 TTh 1:15 – 3:20 pm R-109 4 units

Instructor: Dr. Matt Henes **E-mail:** mthenes@pasadena.edu **Phone:** (626) 585-7456
Office: V-216F **Office Hours:** M-Th 9-9:30 in V-216F; MW 1-2:45 Zoom ID 310 291 0470

Text: "Elementary Linear Algebra: Applications Version," 12th ed. by Anton and Kaul
ISBN 978-1-119-40672-3. Available at the PCC bookstore or as an eText through WileyPLUS

Prerequisite: Math 005B or Math 005BH.

Course description: Vector spaces, linear transformations, determinants, eigenvalues, eigenvectors, solutions of systems of equations, algebra of matrices, inner product spaces, the Gauss-Jordan algorithm, and the Gram-Schmidt algorithm.

Student Learning Outcomes: Upon successful completion of this course you will be able to:
1. Perform operations on vectors, and prove the properties of these operations.
2. Solve a system of linear equations in several unknowns.
3. Analyze a linear transformation from one vector space to another.
4. Investigate the eigentheory of a square matrix.
5. Prove theorems connecting various concepts in Linear Algebra, using appropriate mathematical vocabulary and proof-writing techniques.

Course Content: Lectures, assignments, and assessments will be based on most of chapters 1-8.

Required Materials:

- Pencils
- Eraser(s)
- Lined paper
- 1 ½" (or larger) binder with dividers
- OR in lieu of the above, a tablet device
- Scientific calculator
- Internet access (for the eText)

Homework (10% of your grade): The best way to learn math is to do it. Homework is your opportunity to practice working problems and learn the math you'll see in class. Math tends to build on itself – so one of the best ways to be successful in this course is to do homework regularly and complete assignments on time. Though homework represents a relatively small portion of your overall grade, it is when most of your learning will happen; the time you spend on assignments each week is an investment toward ensuring success on exams. Late homework is discouraged. If something comes up and you're unable to complete an assignment on time, I want you to turn it in when you can. As long as it's submitted before the exam on which the material appears, you'll get some (reduced) credit for it. Your four lowest homework scores will be dropped (this is about 10 percent of the homework assignments for the course).

Exams (60% of your grade): Exams are your opportunity to show what you've learned in this course, so they account for the largest portion of your grade. In this class, we'll have four exams. I've included exam content and dates on the course schedule, and I ask that you please take them into account when planning out your semester. Arranging for make-up tests can be challenging, so optimally, everyone will take exams in class as scheduled. If something comes up, I'll consider make-ups for missed exams on a case-by-case basis. Because exams are intended to reflect what you've learned, and because this course is intended to prepare you for future STEM classes, all exam scores will be included in your final grade.

Comprehensive Final Exam (30% of your grade): Tuesday, June 9, 1:00 – 3:00 pm
The final includes material from throughout the course, so when you prepare for the exam, you'll be able to reinforce the concepts you've learned throughout the term. To pass the class, you are required to take the final exam.

Academic Dishonesty: Academic integrity is important to ensure that the grades I submit are meaningful. Unfortunately, I need to address cheating. Any student caught cheating on an exam will receive a grade deduction for the exam and can possibly receive a score of zero for that exam. I am required to report the violation to the Office of Student Life for potential further action. According to PCC policy, any form of academic dishonesty can be grounds for receiving a grade of F and possibly dismissal from the college. Please make sure you have an official photo ID, as it will be checked during this class to verify that students in the class are those who are officially enrolled.

Calculators: Use of scientific calculators will be allowed on exams. Use of any other electronic device will be considered cheating.

Assigned grades are based on performance on exams and homework, not on extra credit or external factors such as GPA and transfer. Course grades will be no lower than those shown below.
A: 89.5-100% B: 79.5-89.49% C: 72.5-79.49% D: 59.5-72.49 F: below 59.5%

Canvas: Class notes and grades will be recorded on Canvas. Please let me know of any discrepancies or mistakes in posted scores within one week of being posted so I can correct them in a timely manner.

Attendance/Classroom Policies:

- Attend regularly
- Arrive on time
- Don't leave early
- Silence cell phone
- No food or drinks except bottled water
- No visitors (including children)

Additional Information: Tutoring is available at the Math Success Center (R-406), as well as the Zone for athletes (GM-112A). The office of Disabled Student Programs and Services (D-209) provides support for students with documented disabilities; please let me know of any accommodations for which you qualify.

Important Dates:
3/1 Last Day to Add or Drop Without a "W"
3/31 Cesar Chavez Day (no classes)
4/13-18 Spring Break (no classes)
5/15 Last Day to Drop With a "W"
5/25 Memorial Day (no class

Student Performance Objectives

1a. Find the sum of two vectors and a scalar multiple of a vector.
1b. Prove the basic arithmetic properties of vector operations in Euclidean spaces R^n.
1c. Determine if a set, together with an addition and a scalar operation, satisfies all the axioms of a general vector space.
1d. Determine if a bilinear form on a vector space satisfies all the axioms of an inner product space.
1e. Find the inner product of two vectors, the angle between two vectors, and the length of a vector.
1f. Determine if two vectors are orthogonal.
1g. Apply the Gram-Schmidt algorithm to a basis for an inner-product space in order to construct an orthogonal and orthonormal basis for the space.
1h. Find a basis for the orthogonal complement of a subspace in the finite-dimensional case.

2a. Construct the augmented matrix that represents a system of equations.
2b. Prove that row operations do not change the solution set to a system of equations.
2c. Use row operations to find the reduced row echelon form (rref) of a matrix.
2d. Apply the Gauss-Jordan Algorithm to find the rref of a matrix.
2e. Identify the leading and free variables in the rref, and use these to describe all the solutions of the linear system.
2f. Determine if a set of vectors is linearly independent or dependent.
2g. Use the rref to find a basis for the row space, column space, and null space of a matrix and determine its rank and nullity.
2h. Determine if a set of vectors forms a subspace, and if so, find a basis for the subspace and determine the dimension of the subspace.

3a. Determine if a function from one vector space to another satisfies the definition of a linear transformation.
3b. Graph the action of a linear operator on Euclidean 2- or 3-space, including rotations, projections, and reflections.
3c. Perform arithmetic on matrices, including finding the sum or product of two compatible matrices, the transpose of a matrix, and a scalar multiple of a matrix.
3d. Find a basis for the kernel and the range of a linear transformation.
3e. Determine if a linear transformation is injective (one-to-one) or surjective (onto).
3f. Find the inverse of a matrix, and the inverse of its associated linear transformation, or explain why the inverse does not exist.
3g. Use the inverse to solve a matrix equation with a square coefficient matrix.
3h. Construct the matrix of a linear transformation from one finite-dimensional vector space to another, relative to a basis for each space.
3i. Construct the change of basis matrix from one ordered basis to another, in the finite dimensional case.
3j. Solve application problems involving linear transformations on abstract vector spaces.

4a. Find the determinant of a square matrix using permutations, row operations, or a cofactor expansion.
4b. Compute the characteristic polynomial of a square matrix.
4c. Find the eigenvalues of a square matrix, and a basis for each of its eigenspaces, consisting of eigenvectors.
4d. Determine if a square matrix is diagonalizable or not, and if so, apply the diagonalization algorithm.
4e. Prove that a matrix is diagonalizable if and only if it has a complete set of eigenvectors.

4f. Construct an orthogonal matrix that diagonalizes a symmetric matrix.

5a. Prove properties of vector and matrix operations, such as the associative property of matrix multiplication.
5b. Prove that certain special sets of matrices, such as diagonal, triangular, and symmetric matrices, are closed under matrix addition, matrix multiplication, and scalar multiplication, and are thus subspaces.
5c. Prove uniqueness properties, such as the uniqueness of: the zero vector; the inverse of an invertible matrix; the coordinate vector of a given vector relative to a fixed basis; the rref of a matrix.
5d. Prove that the image of a set of independent vectors under an injective (one-to-one) transformation is also an independent set.
5e. Prove properties of permutations.
5f. Prove the effects of row and column operations on the determinant of a matrix.
5g. Prove that the determinant function is multiplicative.

Course Content Outline

I. Euclidean n-Space (R^n)
- A. Vector arithmetic in R^n: addition, scalar multiplication, norm, and linear combinations
- B. Graphing vectors in 2 or 3 dimensions
- C. The span of a set of vectors in R^n
- D. The dot product in R^n
- E. The Cauchy-Schwarz Inequality
- F. Angle and orthogonality in R^n
- G. Systems of linear equations
- H. Elementary row operations on matrices, and their properties
- I. The Gauss-Jordan Algorithm
- J. Linear dependence and independence
- K. The Minimizing Theorem
- L. The Extension Theorem
- M. Subspaces of R^n
- N. Basis for a subspace and the dimension of a subspace
- O. The transpose operation
- P. The four fundamental matrix spaces:
  - (1) row space,
  - (2) column space,
  - (3) null space of a matrix, and
  - (4) null space of its transpose, and their relevance.
- Q. The orthogonal complement of a subspace

II. Linear Transformations of Euclidean Spaces
- A. The standard matrix of a linear transformation
- B. Graphing a linear operator
- C. Dilation, contraction, and shear operators in R^2
- D. Rotations in R^2
- E. Projections and reflections in R^2 and R^3
- F. Arithmetic operations on linear transformations, and their properties
- G. Compositions of linear transformations
- H. Arithmetic operations on matrices, and their properties
- I. Kernel and range of a linear transformation

- J. Injective transformations and surjective transformations
- K. Applying the Gauss-Jordan Algorithm to find the inverse of an invertible matrix
- L. Properties of invertible matrices
- M. Diagonal, triangular, and symmetric matrices, and their properties

III. Abstract Vector Spaces and their Linear Transformations
- A. The axioms of an abstract vector space
- B. Examples of abstract vector spaces, including function spaces and matrix spaces
- C. Linear combinations of sets of vectors, including the infinite case
- D. Subspaces of an abstract vector space; basis and dimension
- E. Linear transformations of abstract vector spaces
- F. Coordinate vectors with respect to a fixed basis
- G. The matrix of a linear transformation with respect to two bases
- H. Kernel and range; injective and surjective linear transformations

IV. The Determinant Function
- A. Permutations and their properties
- B. The definition of the determinant function using permutations
- C. Using row and column operations to compute the determinant
- D. Cofactor expansion
- E. Cramer's Rule

V. Eigentheory of a Square Matrix
- A. Definition of eigenvalue and eigenvector, and their geometric interpretation
- B. The characteristic polynomial
- C. Eigenspaces
- D. Apply the concept of eigenspaces to diagonalize a matrix, when possible.
- E. Change of basis

VI. Inner Product Spaces
- A. Axioms of an inner product space
- B. Angles and orthogonality in inner product spaces
- C. Orthogonal and orthonormal sets
- D. The Gram-Schmidt Algorithm
- E. The orthogonal complement of a subspace of an inner product space
- F. Orthogonal decompositions
- G. Projection operators and their matrices
- H. Orthogonal matrices
- I. Orthogonal diagonalization of symmetric matrices.

VII. Applications of Linear Algebra
- A. Systems of Equations in Science, such as Basic Circuit Analysis or Balancing Chemical Reactions B. The Wronskian of a set of functions
- C. The exponential of a diagonalizable matrix
- D. Finding higher derivatives and integrals of functions found in finite-dimensional spaces preserved by the derivative and extending these ideas to solve linear differential equations with constant coefficients.